How to estimate an Oscillator Drift in Holdover periods

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When dealing with oscillator holdover performance one notices that there is no linear relationship between the frequency offset specifications for example one day of holdover and one year of holdover. This is due to the fact that the drift rate is slightly decreasing as holdover period is increasing. Typically, manufacturers of TCXOs or OCXOs use the MIL-O-55310 Model (MIL-Model) which is given by Eq. 1. to describe the behavior of frequency drift of their oscillators and extrapolate drift values for holdover periods that are outside the scope of their measurements.

$$\Delta f(t) = a \cdot \ln(b \cdot t + 1) \qquad \text{Eq.1.}$$

If we relate the Eq.1. to the nominal oscillator frequency f_0 we get the equation for the dimensionless relative frequency drift which is a more common value as it makes oscillators operating on different frequencies comparable at one glance.

$$\frac{\Delta f(t)}{f_0} = A \cdot \ln(B \cdot t + 1) \qquad \text{Eq.2.}$$

Further the relative frequency offset gives a direct indication of the drift speed [s/s] of a clock based on the nominal frequency (f_0) of that oscillator. The graph below shows a typical curve of relative frequency drift according to the MIL-Model. Please consider this is not a real life data, but an optimized model to predict the oscillator behavior over a long term period.

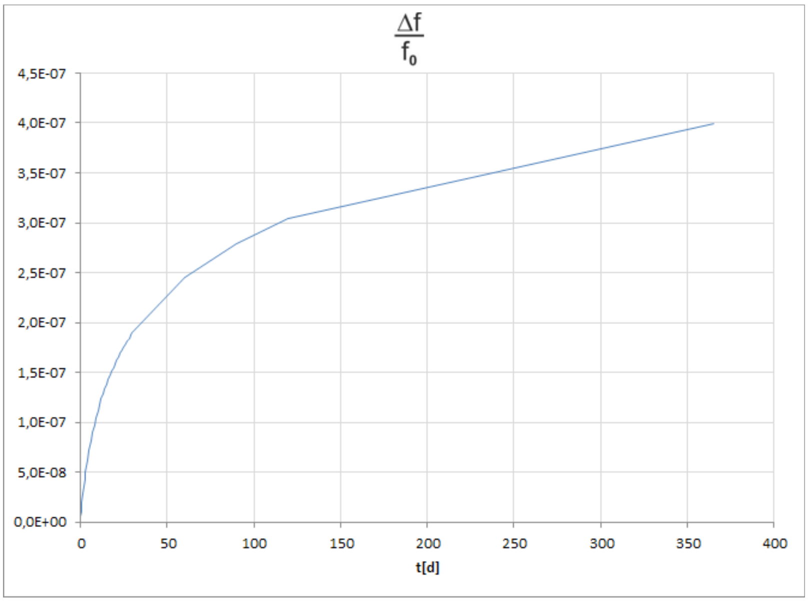

Figure 1: Relative Frequency drift of an oscillator approximated by the MIL-Model curve.

Crystal manufactures typically do not disclose any information about the holdover prediction model such as parameters A and B or any modifications to the model itself. They rather specify two typical aging values, for example after one day and after one year. In this case we try to find an estimate for the relative frequency offset for any holdover period between one day and one year.

Even though it is possible to re-calculate the parameters of the MIL-Model from the specified frequency offset values iteratively, it is a more common approach to approximate a linear frequency drift. The linearized model is gained by applying the tangent close or at t = 0 to the model. As the tangent model goes through the coordinate origin, its computed offsets are always above the modeled values i.e. it represents a worst case value.

The main disadvantage of this method is that it still needs the modeling parameters A and B as the gradient must be computed from the 1st derivative of the MIL-Model. As the oscillator specifications typically give only certain holdover values, we can overcome this by further approximating the gradient from the 1-day holdover value.

$$\frac{\Delta f}{f_0}(t) \approx C_{osc} \cdot t$$

Eq.3.

$$C_{OSC} \approx \frac{\frac{\Delta f_{1\,day}}{f_0}}{86400\,s}$$

This further approximation leads to values that are slightly better than those of the real model as long as holdover period is below 1day. For longer periods the approximated values are increasingly worse than those of the model (see Figure 2).

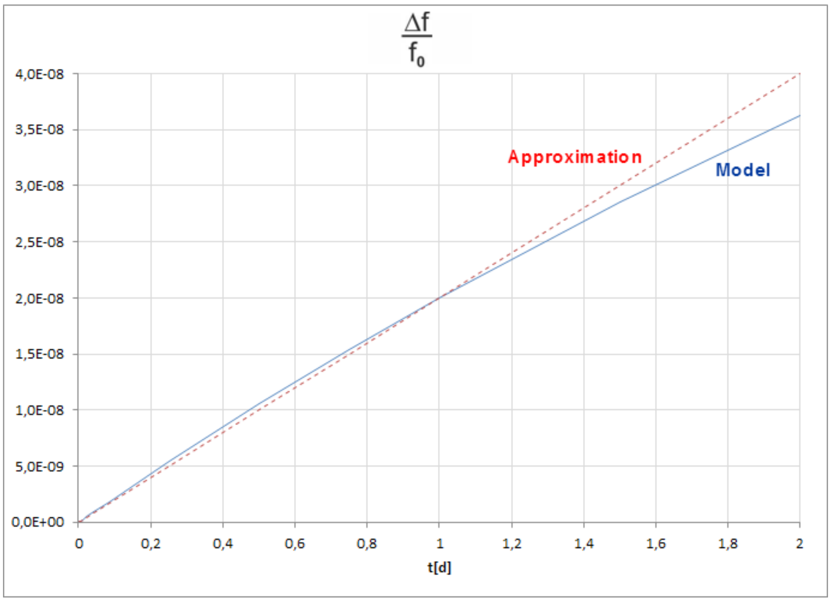

Figure 2: Approximation tangent using gradient based 1-day holdover value (red line); MIL-Model approximation (blue line).

From frequency drift to phase drift

The clock phase drift can be now easily estimated from relative frequency drift by integrating the approximation over the desired holdover period.

$$\varphi \approx \int_0^{t_{holdover}} C_{osc} \cdot t \, dt \approx \frac{1}{2} C_{osc} \cdot t_{holdover}{}^2 \qquad \text{Eq.4.}$$

As the relative frequency shows a linear drift the phase or time offset drifts with the square of the holdover time. Of course this is only an approximation that gives at least the worst case scenario, which is often enough to select the right TCXO or OCXO for an application.

Using Eq.3. and Eq. 4. we can easily calculate some figures for a real oscillator as datasheet values. Assume a real TCXO is specified to drift 1Hz after on day of holdover.

$$\frac{\Delta f_{1day}}{f_0} = \frac{1\,Hz}{1 \cdot 10^7\,Hz} = 1 \cdot 10^{-7} \rightarrow C_{OSC} \approx \frac{1 \cdot 10^{-7}}{86400\,s} \approx \underline{1.157 \cdot 10^{-12}} \qquad \text{Eq.5.}$$

$$\varphi \approx \frac{1.157 \cdot 10^{-12}}{2} \cdot t_{holdover}^{2} \qquad \text{Eq.6.}$$

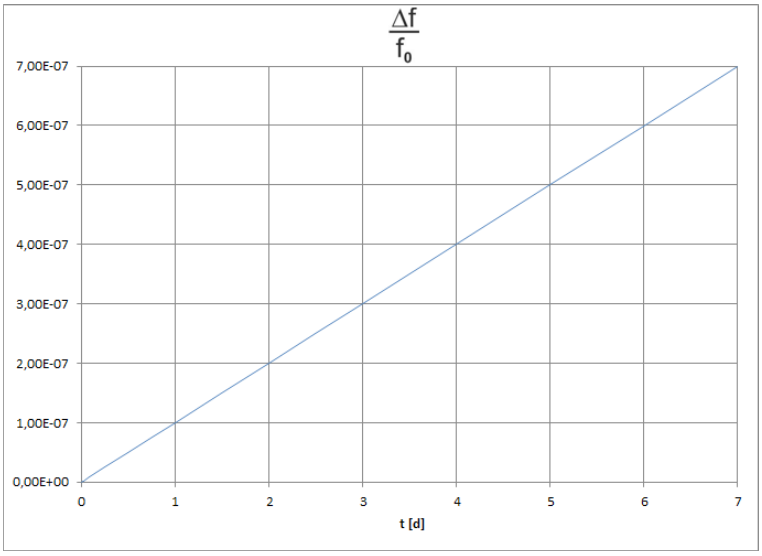

Figure 3: Linear approximation of the freq. drift for TCXO using C_{OSC} calculated by Eq.5.

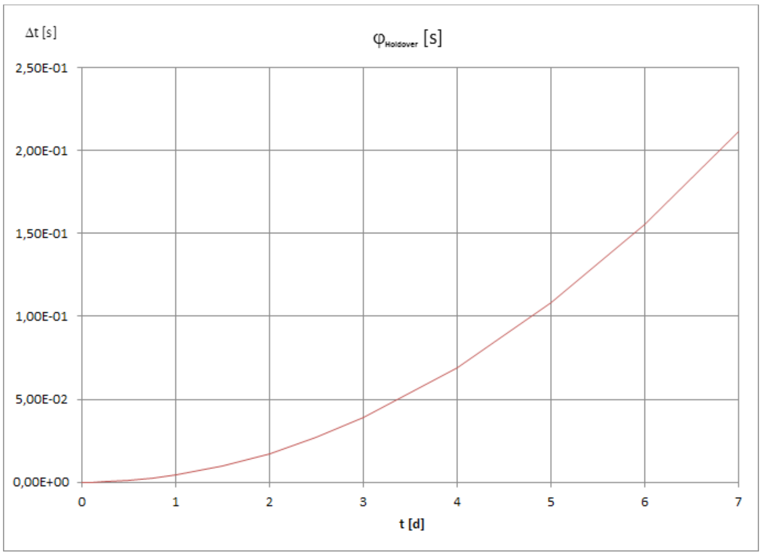

Figure 4: Phase drift estimation for oscillator TCXO calculated by Eq.6.

An example for a real TCXO

A practical example for a real TCXO is shown as follows. The data for this example has been recorded during a qualification process at nearly constant ambient temperature and after several days of initial synchronization, for the stability of the oscillator.

After one day of holdover operation the oscillator shows a frequency offset of $5.7 \cdot 10^{-2}$ Hz which according to Eq.5. gives us a relative frequency offset of $5.72 \cdot 10^{-9}$. This is far below the specification of 1 Hz or $1 \cdot 10^{-7}$ after one day given by the manufacturer. Now the drift rate C_{osc} can be computed approximately to:

$$C_{OSC} \approx \frac{5.72 \cdot 10^{-9}}{86400\, s} \approx \underline{6.19 \cdot 10^{-14}} \frac{1}{s} \qquad \text{Eq.7.}$$

$$\varphi \approx \frac{6.19 \cdot 10^{-14}}{2} \cdot t_{hol\,dover}^{\;2} \qquad \text{Eq.8.}$$

Recorded phase measurements as well as its approximation using frequency offset for one day from Eq.7. are plotted in Figure 5. As expected the approximation shows better values than the real measurement as long as the holdover time is below one day. Above one day approximation becomes increasingly worse however it is still sufficient for to give an estimate for choosing the right oscillator type. Better approximations can be achieved by taking the whole dataset into consideration using least squares fit.

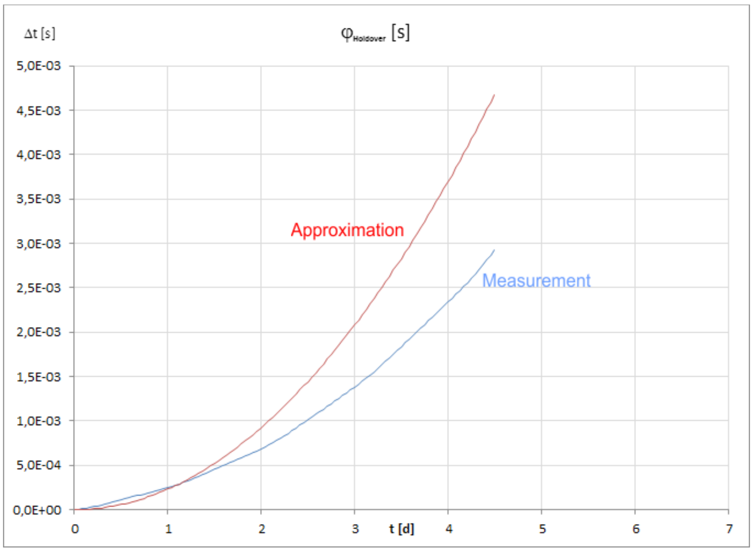

Figure 5: Real phase measurements for TCXO oscillator (blue line) and approximated ones using the frequency drift offset for one day holdover (red line). Note that approximation shows slightly better values than real measurements for holdover below one day and increasingly worse for holdover over one day.